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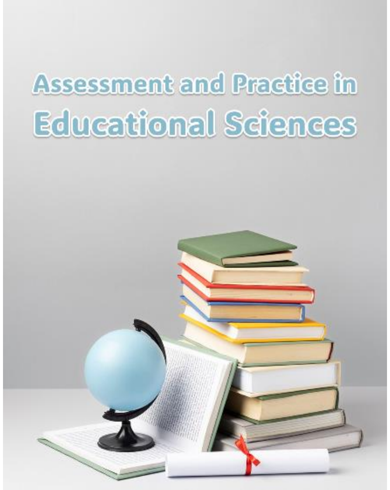

The Analysis of the Role of Integrated Critical Thinking Instruction in Improving Mathematics Performance among Secondary School Students: A Mixed-Methods Study in Tehran Schools

1. Ali. Rahmati: Department of Mathematics Education, SR.C, Islamic Azad University, Tehran, Iran

2. Mohammad Hassan. Behzadi*: Department of Statistics, SR.C, Islamic Azad University, Tehran, Iran. (Email: behzadi@srbiau.ac.ir)

3. Mohsen. Rostamy Malkhalifeh: Department of Computer Science - Soft Computing and Artificial Intelligence, SR.C, Islamic Azad University, Tehran, Iran

4. Ahmad. Shahvarani Semnani: Department of Mathematics Education, SR.C, Islamic Azad University, Tehran, Iran.

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ABSTRACT

The present study aimed to analyze the role of integrated critical thinking instruction in improving mathematics performance and critical thinking skills among secondary school students in Tehran using an exploratory sequential mixed-methods approach. This study employed an applied mixed-methods design with an exploratory sequential approach. In the qualitative phase, data were collected through semi-structured interviews with 20 mathematics education experts and experienced secondary school teachers, classroom observations, and document analysis. The qualitative findings were analyzed using thematic analysis in MAXQDA software to identify the major components of critical thinking in mathematics education. In the quantitative phase, a quasi-experimental pretest-posttest design with control and experimental groups was implemented. The participants included 120 tenth-grade students selected from four secondary school classes in Tehran, with two classes assigned to the experimental group and two classes to the control group. The intervention consisted of eight 90-minute instructional sessions integrating critical thinking strategies into mathematics teaching. Data collection instruments included a researcher-made mathematics performance test. Quantitative data were analyzed using descriptive statistics and analysis of covariance (ANCOVA) in SPSS software. The qualitative findings identified major themes including analytical reasoning, reflective thinking, conceptual understanding, inquiry-based learning, collaborative reasoning, and real-life application of mathematics. Quantitative findings demonstrated that integrated critical thinking instruction had a statistically significant effect on students' mathematics performance and critical thinking abilities. ANCOVA results revealed significant differences between the experimental and control groups in mathematics performance, $F(1,117)=73.92$, $p<0.001$, $\eta^2=0.39$, after controlling for pretest differences. The findings indicated that integrating critical thinking strategies into mathematics instruction significantly enhances students' mathematics achievement, analytical reasoning, conceptual understanding, and reflective thinking abilities. The study supports the effectiveness of learner-centered and inquiry-based instructional approaches in improving mathematics education and highlights the importance of incorporating critical thinking components into secondary school curricula and classroom practices.

Keywords: Critical Thinking, Mathematics Education, Mathematics Performance, Secondary School Students

Introduction

Mathematics education has long been recognized as one of the fundamental pillars of educational systems because of its essential role in developing logical reasoning, analytical thinking, and problem-solving abilities among learners. In contemporary educational discourse, mathematics is no longer perceived merely as a collection of formulas and computational procedures; rather, it is increasingly viewed as a dynamic intellectual domain that fosters reasoning, creativity, and critical inquiry. Educational policymakers and curriculum designers have therefore emphasized the necessity of transforming mathematics instruction from traditional teacher-centered approaches toward learner-centered and cognitively engaging pedagogies that cultivate higher-order thinking skills (1, 2). Within this context, critical thinking has emerged as one of the most significant competencies required for effective learning and successful participation in modern societies.

Critical thinking refers to the capacity to analyze information logically, evaluate evidence systematically, interpret arguments critically, and make reasoned judgments based on reflective thought processes. In educational settings, critical thinking enables students to move beyond rote memorization and passive learning toward active exploration, conceptual understanding, and meaningful knowledge construction. Contemporary educational systems increasingly recognize that students who possess strong critical thinking skills are more capable of solving complex problems, adapting to new situations, and engaging productively in collaborative and interdisciplinary learning environments (3, 4). Consequently, integrating critical thinking into mathematics education has become a major priority for researchers, educators, and curriculum developers worldwide.

The importance of critical thinking in mathematics education becomes particularly evident when considering the nature of mathematical reasoning itself. Mathematics requires learners to analyze relationships, construct logical arguments, identify patterns, evaluate alternative solutions, and justify conclusions through systematic reasoning processes. These cognitive activities align closely with the essential dimensions of critical thinking, indicating that mathematics classrooms can serve as highly effective environments for cultivating analytical and reflective thinking skills. However, despite the theoretical compatibility between mathematics and critical thinking, instructional practices in many educational systems continue to rely heavily on procedural teaching methods emphasizing memorization and repetitive exercises rather than conceptual understanding and reasoning-based learning (5, 6).

Traditional mathematics instruction often positions students as passive recipients of knowledge, limiting opportunities for inquiry, discussion, exploration, and reflective analysis. Such approaches may negatively influence students' motivation, creativity, and problem-solving abilities, ultimately reducing the effectiveness of mathematics learning. Research has shown that instructional methods focused exclusively on procedural accuracy may fail to develop students' deeper conceptual understanding and higher-order cognitive abilities (7, 8). As educational systems increasingly prioritize competency-based learning and twenty-first-century skills, there is growing recognition that mathematics instruction must incorporate pedagogical strategies capable of fostering active engagement, analytical reasoning, and critical reflection.

Constructivist learning theories provide a significant theoretical foundation for integrating critical thinking into mathematics instruction. According to constructivist perspectives, knowledge is actively constructed by learners through interaction with ideas, experiences, and social contexts rather than transmitted directly from teacher to student. Constructivist instructional approaches encourage learners to engage in questioning, collaborative discussion, reflective analysis, and authentic problem solving, thereby promoting deeper conceptual understanding and cognitive flexibility. A systematic review conducted by Arega and Hunde demonstrated that constructivist instructional approaches significantly enhance students' learning outcomes, engagement, and critical reasoning abilities across educational contexts (1). These findings suggest that instructional models integrating critical thinking within mathematics education may substantially improve students' academic performance and cognitive development.

The integration of active learning strategies into mathematics education has also received considerable scholarly attention in recent years. Active learning approaches emphasize student participation, collaborative interaction, inquiry-based activities, and experiential engagement in the learning process. Such approaches encourage learners to become active constructors of knowledge rather than passive recipients of information. Yee et al. highlighted that active learning environments promote deeper understanding, increased motivation, and greater intellectual engagement among students by encouraging meaningful interaction with content and peers (9). Similarly, Aliyu et al. found that innovative learning strategies designed to enhance student engagement in mathematics instruction significantly improved learners' participation and conceptual comprehension (10). These findings reinforce the argument that integrating critical thinking strategies within mathematics classrooms may create more effective and intellectually stimulating learning environments.

Another important dimension of contemporary mathematics education involves the application of mathematics to authentic and real-world contexts. Researchers have argued that mathematics instruction becomes more meaningful and engaging when students can connect mathematical concepts to practical situations and real-life experiences. Han et al. emphasized the importance of using real-world contexts in instructional materials to strengthen students' conceptual understanding and problem-solving abilities (5). Similarly, Chien et al. demonstrated that engineering and STEM-based design projects foster collaborative reasoning, creativity, and analytical thinking among learners by integrating mathematical learning with authentic problem-solving experiences (11). These perspectives suggest that mathematics education should not be limited to abstract procedural exercises but should instead encourage students to apply mathematical reasoning within realistic and interdisciplinary contexts.

Technological advancements have further transformed contemporary approaches to mathematics instruction and critical thinking development. Digital technologies, artificial intelligence, and interactive educational software have created new opportunities for supporting analytical reasoning and personalized learning processes. Zhou et al. explored the role of ChatGPT-facilitated scaffolding in undergraduate mathematical problem solving and found that artificial intelligence tools can enhance students' reasoning processes, reflective thinking, and problem-solving performance when integrated effectively into instructional environments (12). Likewise, Heidarpour et al. emphasized the growing role of artificial intelligence in teaching soft skills and critical thinking, arguing that technology-enhanced learning environments can facilitate higher-order cognitive development and adaptive learning experiences (13). Interactive educational software has also been shown to positively influence students' mathematics achievement and engagement by providing dynamic and learner-centered instructional experiences (14).

The effectiveness of critical thinking integration within mathematics education also depends substantially on teacher preparation and pedagogical competence. Teachers play a central role in designing learning environments that encourage inquiry, reasoning, and reflective thinking. Effective mathematics teachers must possess both strong pedagogical content knowledge and the ability to facilitate meaningful classroom interaction and cognitive engagement. Habiyaremye et al. highlighted the importance of pedagogical content knowledge in implementing competency-based mathematics instruction and promoting conceptual understanding among learners (15). Similarly, Saleh and AlAli emphasized that mathematics teacher preparation programs should incorporate instructional frameworks supporting active learning, analytical reasoning, and critical inquiry (8). The success of integrated critical thinking instruction therefore depends not only on curriculum design but also on teachers' professional competence and instructional practices.

Curriculum innovation and educational reform initiatives worldwide increasingly emphasize the integration of creativity, critical thinking, and problem-solving skills within mathematics education. Bingol and Özyaprak argued that differentiated curriculum and instructional practices can foster mathematical creativity and learner motivation by encouraging exploration, flexibility, and higher-order reasoning (2). Likewise, Matorevhu emphasized that curriculum innovation aimed at preparing students for industrialization and modern workforce demands requires educational systems to prioritize critical thinking and adaptive problem-solving abilities (16). These perspectives align with broader global educational trends emphasizing the development of transferable cognitive competencies necessary for success in rapidly changing social and technological environments.

Despite the increasing emphasis on critical thinking in educational policy and research, many schools continue to face significant challenges in implementing effective critical thinking instruction within mathematics classrooms. These challenges may include limited teacher preparation, reliance on examination-oriented instructional methods, insufficient instructional resources, rigid curricula, and inadequate opportunities for collaborative and inquiry-based learning. Educational systems influenced by traditional pedagogical paradigms often prioritize procedural performance over analytical understanding, thereby restricting students' opportunities to develop higher-order thinking abilities (6, 17). In many contexts, mathematics classrooms remain heavily teacher-centered, with limited emphasis on discussion, reflection, and exploratory reasoning.

Research has further indicated that critical thinking development requires carefully structured instructional interventions rather than isolated classroom activities. Talanquer et al. argued that effective learning occurs within interconnected systems involving classroom interaction, instructional design, cognitive processes, and sociocultural contexts (18). Consequently, the integration of critical thinking into mathematics instruction requires comprehensive and systematic instructional models capable of aligning curriculum objectives, teaching practices, assessment methods, and student engagement strategies. Rezaee and Saleh similarly emphasized that critical thinking development depends on the deliberate incorporation of reflective and analytical components within educational materials and instructional frameworks (19).

The need for integrated critical thinking instruction is particularly significant in secondary education, where students encounter increasingly abstract mathematical concepts and more complex reasoning demands. Secondary school students often struggle with conceptual understanding, analytical problem solving, and mathematical argumentation when instruction focuses primarily on procedural memorization. Research indicates that instructional strategies emphasizing inquiry, reflection, collaboration, and conceptual analysis can substantially improve students' mathematical achievement and intellectual engagement (20, 21). Furthermore, integrating critical thinking into mathematics education may contribute not only to academic improvement but also to the development of broader cognitive and social competencies necessary for lifelong learning.

In the Iranian educational context, concerns regarding the effectiveness of traditional mathematics instruction and students' limited critical thinking abilities have intensified in recent years. Educational researchers and policymakers have increasingly emphasized the necessity of curriculum reform, active learning, and competency-based instruction to improve educational quality and learner outcomes. Mostafavi Eskouei et al. stressed the importance of teaching critical thinking and problem-solving skills within school curricula to prepare students for contemporary social and professional challenges (3). Similarly, Halajjan et al. highlighted the role of curriculum planning strategies in optimizing critical thinking and problem-solving abilities among students through learner-centered and cognitively engaging instructional approaches (4). These developments suggest an urgent need for empirical research examining the practical effectiveness of integrated critical thinking instruction in Iranian mathematics classrooms.

Although previous studies have examined active learning, constructivist instruction, technology integration, and mathematical reasoning separately, relatively few studies have investigated the integrated role of critical thinking instruction in improving mathematics performance among secondary school students using a comprehensive mixed-methods approach. Moreover, limited research has explored how qualitative insights derived from classroom observations and teacher experiences can inform the design and validation of instructional interventions aimed at enhancing critical thinking within mathematics education. Addressing this gap is particularly important for developing evidence-based instructional models that are contextually relevant, pedagogically feasible, and educationally effective.

Therefore, the present study aimed to analyze the role of integrated critical thinking instruction in improving mathematics performance among secondary school students through a mixed-methods exploratory sequential study conducted in secondary schools in Tehran.

Methods and Materials

This study was applied in terms of purpose and employed a mixed-methods approach using an exploratory sequential design. The qualitative phase was conducted first to identify and conceptualize the major components of critical thinking in mathematics education, while the quantitative phase was designed to validate the extracted model and examine its effectiveness in improving students' mathematics performance. The integration of qualitative and quantitative methods enabled the researchers to obtain a comprehensive understanding of the phenomenon under investigation and to evaluate the practical applicability of the proposed instructional model in authentic educational settings.

In the qualitative phase, data were collected through semi-structured interviews with mathematics education specialists and experienced secondary school mathematics teachers, as well as through direct classroom observations of teaching practices. The purpose of this phase was to explore the dimensions and instructional manifestations of critical thinking within mathematics classrooms and to identify pedagogical strategies that could enhance students' analytical and reasoning abilities. The interviews focused on teachers' perceptions of critical thinking, instructional barriers, classroom interactions, mathematical reasoning processes, and the role of active learning in mathematics achievement. Observational data were used to examine the actual implementation of teaching strategies and student engagement in classroom activities. The collected qualitative data were transcribed, coded, and categorized into themes representing the key components of critical thinking in mathematics education.

The qualitative population included university experts in mathematics education, experienced secondary school mathematics teachers, and mathematics classrooms at the upper secondary level. Using purposive sampling, 20 participants were selected based on specific inclusion criteria, including possession of at least ten years of teaching experience, an academic background related to mathematics education, and research or professional experience in mathematics pedagogy. The participants represented different school types, including public schools, non-profit schools, and gifted schools, and reflected a variety of instructional approaches such as traditional, active, and blended teaching methods. Furthermore, teachers with different levels of teaching experience and from both urban and rural educational settings were included in order to ensure diversity of perspectives and instructional contexts. Classroom observations were conducted across several mathematics classes characterized by varying combinations of teachers, school environments, and dominant instructional approaches. These observations provided contextual insights into how critical thinking skills were fostered or constrained within mathematics instruction.

The findings of the qualitative phase were subsequently utilized in the quantitative phase to develop research instruments, formulate hypotheses, and design the integrated instructional model of critical thinking in mathematics education. In the quantitative phase, the identified components were empirically examined in order to determine their validity and effectiveness in improving mathematics achievement among secondary school students.

The quantitative population consisted of all tenth-grade students enrolled in upper secondary schools in Tehran during the 2023–2024 academic year, secondary school mathematics teachers, and upper secondary schools within the city. Based on Cochran's sampling formula and using multistage cluster sampling, an initial sample size of 380 students was estimated. However, for the experimental implementation of the study, four classes consisting of 30 students each were selected, resulting in a total sample of 120 students. Two classes were assigned to the experimental group and two classes to the control group. Additionally, four mathematics teachers participated in the implementation process, with one teacher assigned to each class.

The intervention program consisted of eight instructional sessions, each lasting 90 minutes and conducted twice per week. The sessions were implemented simultaneously across the four classes by the participating teachers. In the control groups, mathematics instruction was delivered through conventional teaching methods commonly used in secondary schools. In contrast, the experimental groups received mathematics instruction through an integrated critical thinking approach designed based on the findings of the qualitative phase. This instructional model emphasized analytical reasoning, problem-solving, conceptual understanding, questioning strategies, reflective thinking, collaborative discussion, and the application of mathematical concepts in authentic situations.

At the beginning of the intervention, all students completed a mathematics performance pretest in order to assess their baseline levels of mathematical achievement and critical thinking ability. Following the completion of the instructional program, the same instruments were administered again as posttests. The comparison of pretest and posttest scores between the experimental and control groups enabled the researchers to evaluate the effectiveness of the integrated critical thinking model on students' mathematics performance and critical thinking development.

Overall, the exploratory sequential mixed-methods design allowed the qualitative findings to guide the development of the quantitative phase. The qualitative phase functioned as a foundational exploratory stage that provided conceptual and contextual data necessary for designing the instructional intervention and measurement tools, whereas the quantitative phase served to validate and examine the effectiveness of the identified components under real classroom conditions. Therefore, the two phases were interconnected and complementary, and the overall success of the study depended on the effective integration of qualitative insights into the quantitative implementation process.

Data collection in this study was conducted using both library-based and field-based methods. In the library phase, relevant scientific resources including scholarly articles, dissertations, books, and previous national and international studies related to critical thinking and mathematics education were systematically reviewed. Academic databases and reputable scientific sources were consulted to establish the theoretical foundations of the study and support the development of the research framework.

Field data collection involved the use of multiple instruments, a mathematics performance test, semi-structured interviews, and classroom observation checklists. The use of diverse instruments enabled methodological triangulation and enhanced the credibility and comprehensiveness of the findings.

The mathematics performance test was specifically developed by the researchers to assess students' abilities in conceptual understanding, practical application, and analytical reasoning in mathematics. The test consisted of 15 items designed based on the selected chapters of the tenth-grade mathematics curriculum. The items covered several dimensions of mathematical learning, including multi-step problem solving, analysis of numerical and functional relationships, and the application of mathematical concepts to real-life situations.

The test included three categories of questions. The first category focused on conceptual understanding and included items assessing students' comprehension and definition of fundamental mathematical concepts. The second category addressed applied mathematical skills and evaluated students' ability to use mathematical concepts in practical and everyday contexts.

The third category consisted of analytical and reasoning-based problems requiring multi-step reasoning, justification, and critical analysis of mathematical claims and patterns.

To establish content validity, the mathematics performance test was reviewed and approved by five experts in mathematics education and active teaching methodologies. These experts evaluated the extent to which the test items aligned with curriculum objectives, critical thinking components, and the intended educational outcomes. Each item was assessed for conceptual clarity, relevance, instructional alignment, and discriminatory power in identifying students' mathematical reasoning abilities.

The reliability of the mathematics performance test was examined using Cronbach's alpha coefficient. The instrument was piloted on a sample of 30 students prior to the main implementation of the study. The obtained Cronbach's alpha coefficient was 0.81, indicating an acceptable level of internal consistency and reliability for measuring mathematics performance.

Semi-structured interviews conducted during the qualitative phase explored participants' perspectives regarding instructional strategies, student engagement, classroom interaction, and barriers to critical thinking development in mathematics education. Furthermore, classroom observation checklists were employed to document instructional behaviors, student participation patterns, questioning techniques, collaborative learning activities, and opportunities for analytical reasoning during mathematics instruction.

The analysis of qualitative and quantitative data was conducted using specialized software appropriate for each phase of the study. In the qualitative phase, the data obtained from interviews and classroom observations were analyzed through thematic analysis using MAXQDA software. The analysis process involved several stages, including transcription of interviews, initial coding, categorization of codes, identification of themes, and interpretation of thematic relationships. Open coding was first applied to extract meaningful units from the data, followed by axial coding to establish relationships among categories. Finally, selective coding was conducted to identify the central themes representing the key components of critical thinking in mathematics education. The resulting themes formed the conceptual basis for the design of the integrated instructional model and the quantitative research instruments.

In the quantitative phase, data analysis was performed using SPSS software. Descriptive statistical methods including means, standard deviations, frequency distributions, tables, and graphical representations were used to summarize the characteristics of the data and participants. Inferential statistical analyses were subsequently employed to test the research hypotheses and examine the effectiveness of the intervention.

To evaluate the impact of integrated critical thinking instruction on students' mathematics performance and critical thinking skills, analysis of covariance (ANCOVA) was used. This statistical procedure enabled the researchers to compare the posttest scores of the experimental and control groups while controlling for pretest differences. Prior to conducting ANCOVA, assumptions such as normality of data distribution, homogeneity of variances, linearity, and homogeneity of regression slopes were examined to ensure the appropriateness of the analysis. The results of the inferential analyses were interpreted in relation to the study objectives and compared with findings from previous research in mathematics education and critical thinking instruction. The combination of qualitative thematic findings and quantitative statistical evidence provided a comprehensive understanding of the effectiveness and applicability of the proposed integrated instructional model in secondary school mathematics education.

Findings and Results

The findings of the present study are reported in two sections corresponding to the qualitative and quantitative phases of the exploratory sequential mixed-methods design. Initially, the demographic characteristics of the participants are presented, followed by the findings derived from the qualitative thematic analysis and the quantitative statistical analyses. The integration of both qualitative and quantitative findings provided a comprehensive understanding of the effectiveness of integrated critical thinking instruction in improving mathematics performance among secondary school students.

In the qualitative phase, 20 mathematics teachers and specialists participated in the semi-structured interviews and classroom observations. Among the participants, 11 teachers were male and 9 were female. Regarding teaching experience, 6 participants had between 1 and 5 years of experience, 7 participants had between 6 and 15 years of experience, and 7 participants had more than 16 years of teaching experience. In terms of school type, the participants represented public schools, non-profit schools, and gifted schools. Furthermore, the participants utilized different instructional approaches, including traditional, active, and blended teaching methods. The diversity of the participants and educational contexts increased the depth and richness of the qualitative findings.

In the quantitative phase, 120 tenth-grade students participated in the study. Sixty students were assigned to the experimental group and sixty students to the control group. The mean age of the participants was 15.87 years (SD = 0.64). Of the total participants, 58 students were female and 62 students were male. The demographic characteristics indicated that the experimental and control groups were relatively equivalent in terms of age, gender distribution, and educational background prior to the implementation of the intervention.

Table 1. Themes and Coding Results of the Qualitative Phase Related to Critical Thinking Components in Mathematics Education

Main Theme	Subtheme	Sample Codes	Frequency
Analytical Reasoning	Multi-step problem solving	Logical sequencing, decomposition of problems, analytical comparison	31
Reflective Thinking	Self-evaluation during problem solving	Reflection on mistakes, revision of solutions, monitoring reasoning process	27
Conceptual Understanding	Deep comprehension of mathematical concepts	Understanding relationships, conceptual interpretation, abstraction	29
Question-Oriented Learning	Inquiry-based classroom interaction	Student questioning, exploratory discussion, guided inquiry	24
Collaborative Learning	Group-based mathematical reasoning	Peer discussion, cooperative problem solving, idea sharing	22
Real-Life Application	Applying mathematics to authentic contexts	Daily-life examples, practical modeling, contextual reasoning	26
Creative Mathematical Thinking	Flexible solution generation	Alternative strategies, innovation in reasoning, divergent thinking	19
Argumentation Skills	Mathematical justification and defense	Evidence-based explanation, proof construction, logical defense	21
Active Student Engagement	Participation in classroom discourse	Interactive activities, discussion participation, classroom involvement	30
Teacher Facilitation Strategies	Instructional support for critical thinking	Scaffolding, guided discovery, feedback-oriented teaching	28

The findings presented in Table 1 demonstrate that the qualitative analysis identified several interconnected dimensions of critical thinking within mathematics education. Among the extracted themes, analytical reasoning and active student engagement showed the highest frequencies, indicating that teachers and experts considered these dimensions to be central to improving students' mathematical understanding and performance. Participants repeatedly emphasized that students achieved deeper learning when they were encouraged to analyze mathematical relationships, compare alternative solutions, and justify their reasoning processes. Reflective thinking and conceptual understanding were also identified as major components influencing mathematics achievement, particularly when students engaged in self-monitoring and conceptual interpretation rather than rote memorization. Moreover, the findings highlighted the importance of inquiry-based learning, collaborative discussion, and real-life application of mathematical concepts in fostering critical thinking abilities. Teachers consistently reported that classrooms characterized by active participation, guided questioning, and flexible problem-solving environments produced higher levels of student motivation and analytical performance. The thematic analysis ultimately formed the conceptual foundation for the development of the integrated instructional model implemented during the quantitative phase of the study.

Table 2. Integrated Critical Thinking Instructional Intervention Protocol

Session	Instructional Focus	Main Activities	Expected Learning Outcome
Session 1	Introduction to critical thinking in mathematics	Orientation, discussion about reasoning and mathematical thinking	Familiarity with critical thinking concepts
Session 2	Analytical problem-solving strategies	Multi-step mathematical problems and guided analysis	Improvement in analytical reasoning
Session 3	Conceptual understanding of mathematical relationships	Concept mapping and conceptual discussions	Deep conceptual comprehension
Session 4	Questioning and inquiry-based learning	Student-generated questions and exploratory dialogue	Enhancement of inquiry skills
Session 5	Collaborative mathematical reasoning	Group problem-solving activities and peer interaction	Development of cooperative reasoning
Session 6	Real-life application of mathematics	Solving contextual and authentic mathematical problems	Application of mathematics to daily situations
Session 7	Reflective thinking and self-evaluation	Error analysis and reflective learning exercises	Strengthening reflective judgment
Session 8	Integrated reasoning and mathematical argumentation	Presentation and defense of mathematical solutions	Development of argumentation and justification skills

The intervention protocol presented in Table 2 illustrates the structured implementation of the integrated critical thinking instructional model across eight instructional sessions. Each session was specifically designed to target one or more dimensions of critical thinking identified during the qualitative phase of the study. The intervention gradually moved from introductory conceptual discussions toward more advanced reasoning and argumentation activities. Early sessions focused primarily on helping students recognize the role of critical thinking in mathematics and develop analytical problem-solving strategies. Subsequent sessions emphasized conceptual understanding, inquiry-oriented learning, and collaborative reasoning processes. The incorporation of real-life mathematical applications enabled students to perceive mathematics as a meaningful and functional discipline rather than merely a collection of abstract procedures. In the final sessions, students engaged in reflective evaluation and mathematical argumentation, which encouraged them to justify solutions logically and critically examine alternative approaches. Classroom observations conducted during the intervention indicated that students in the experimental groups progressively demonstrated greater participation, confidence, and engagement in mathematical discussions compared with students in the control groups. Overall, the instructional protocol provided a coherent and systematic framework for integrating critical thinking skills into secondary mathematics education.

Table 3. Descriptive Statistics of Mathematics Performance Scores by Group and Time

Variable	Group	N	Pretest Mean	Pretest SD	Posttest Mean	Posttest SD
Mathematics Performance	Experimental	60	14.82	2.91	21.47	2.38
Mathematics Performance	Control	60	15.04	2.76	16.18	2.57

As shown in Table 3, the pretest mean scores of mathematics performance were close in the experimental and control groups, indicating that the two groups were relatively comparable before the intervention. The experimental group obtained a pretest mean score of 14.82 with a standard deviation of 2.91, while the control group obtained a pretest mean score of 15.04 with a standard deviation of 2.76. After the implementation of the integrated critical thinking instructional model, the posttest mean score of the experimental group increased to 21.47 with a standard deviation of 2.38. In contrast, the control group showed only a limited increase, reaching a posttest mean score of 16.18 with a standard deviation of 2.57. These descriptive findings indicate that students who received mathematics instruction through the integrated critical thinking model demonstrated a considerably greater improvement in mathematics performance than students who received conventional instruction.

Table 4. ANCOVA Results for the Effect of Integrated Critical Thinking Instruction on Mathematics Performance

Source	Sum of Squares	df	Mean Square	F	Sig.	Eta Squared
Pretest Mathematics Performance	118.43	1	118.43	12.57	0.001	0.10
Group	694.81	1	694.81	73.92	0.001	0.39
Error	1098.52	117	9.39	—	—	—
Corrected Total	1911.76	119	—	—	—	—

As presented in Table 4, analysis of covariance was conducted to examine the effectiveness of integrated critical thinking instruction on students' mathematics performance while controlling for pretest scores. The results showed that the effect of the pretest was statistically significant, $F(1,117) = 12.57$, $p = 0.001$, indicating that students' initial level of mathematics performance was meaningfully related to their posttest performance. More importantly, after controlling for the pretest effect, the effect of group membership was statistically significant, $F(1,117) = 73.92$, $p = 0.001$. This finding indicates that there was a significant difference between the experimental and control groups in posttest mathematics performance. The eta squared value for the group effect was 0.39, showing that 39% of the variance in posttest mathematics performance was explained by the instructional intervention. Therefore, the integrated critical thinking instructional model had a strong and statistically significant effect on improving mathematics performance among tenth-grade students.

Discussion and Conclusion

The present study aimed to analyze the role of integrated critical thinking instruction in improving mathematics performance among secondary school students using an exploratory sequential mixed-methods approach. The findings of both the qualitative and quantitative phases demonstrated that integrating critical thinking strategies into mathematics instruction significantly enhanced students' mathematics achievement, conceptual understanding, analytical reasoning, reflective thinking, and classroom engagement. The qualitative findings identified several central dimensions of effective critical thinking instruction in mathematics education, including analytical reasoning, inquiry-based learning, collaborative interaction, conceptual understanding, reflective evaluation, real-life application of mathematical concepts, and mathematical argumentation. The quantitative findings further confirmed that students who participated in the integrated critical thinking instructional program demonstrated significantly higher mathematics performance and critical thinking scores compared with students who received traditional mathematics instruction.

One of the major findings of the study was that integrated critical thinking instruction substantially improved students' mathematics performance. The ANCOVA results indicated significant differences between the experimental and control groups after controlling for pretest scores, suggesting that the instructional intervention had a meaningful educational impact. This finding supports the view that mathematics learning becomes more effective when students actively engage in reasoning, inquiry, and conceptual analysis rather than relying solely on procedural memorization. In traditional mathematics classrooms, students often focus on reproducing formulas and algorithms without developing a deep understanding of mathematical relationships. In contrast, the integrated instructional model used in the present study encouraged students to analyze problems systematically, evaluate multiple solution pathways, and justify their reasoning processes, thereby promoting deeper cognitive processing and improved learning outcomes.

These findings are consistent with the constructivist perspective emphasizing active knowledge construction through meaningful engagement and reflective inquiry. Arega and Hunde reported that constructivist instructional approaches significantly improve students' learning outcomes, critical reasoning abilities, and classroom participation by encouraging active cognitive involvement in the learning process (1). Similarly, Yee et al. highlighted that active learning environments promote deeper conceptual understanding and intellectual engagement because students participate actively in discussion, collaboration, and problem-solving activities rather than remaining passive recipients of information (9). The present findings therefore reinforce the argument that mathematics education should move beyond teacher-centered instruction toward cognitively engaging and learner-centered pedagogical models.

Another important finding of the study involved the significant improvement in students' critical thinking abilities following the implementation of the integrated instructional intervention. Students in the experimental groups demonstrated substantial progress in analytical reasoning, reflective judgment, conceptual interpretation, and mathematical argumentation compared with students in the control groups. This finding suggests that critical thinking skills can be effectively developed through structured instructional experiences integrated within mathematics education. The intervention protocol designed in the present study intentionally incorporated activities such as inquiry-based discussion, collaborative reasoning, real-life problem solving, and reflective evaluation, all of which contributed to the enhancement of students' higher-order thinking abilities.

This finding aligns with the results reported by Mostafavi Eskouei et al., who emphasized the importance of explicitly teaching critical thinking and problem-solving skills within school curricula to foster analytical and reflective thinking among learners (3). Likewise, Halajjan et al. concluded that curriculum planning strategies emphasizing critical thinking and problem-solving significantly contribute to students' cognitive development and intellectual flexibility (4). The consistency between these studies and the present findings indicates that critical thinking development is most effective when instructional activities intentionally encourage learners to question assumptions, analyze relationships, and engage in reflective reasoning processes.

The qualitative findings of the present study further revealed that analytical reasoning and active student engagement were among the most prominent components identified by teachers and educational experts. Participants repeatedly emphasized that students' understanding of mathematics improved considerably when they were encouraged to explain their reasoning, discuss alternative solutions, and participate actively in classroom interaction. These findings support the argument that mathematical understanding is strengthened through dialogue, inquiry, and collaborative learning experiences rather than through isolated individual practice alone. In this regard, Aliyu et al. demonstrated that instructional strategies designed to increase student engagement in mathematics classrooms positively influence learners' participation, motivation, and conceptual understanding (10). Similarly, Bingol and Özyaprak argued that differentiated and creativity-oriented instructional approaches foster mathematical motivation and intellectual engagement by encouraging flexible and exploratory learning experiences (2).

The significant role of collaborative learning observed in the present study may also be explained through sociocultural perspectives on learning, which emphasize the importance of social interaction in cognitive development. Students in the experimental groups frequently engaged in peer discussion, group problem solving, and cooperative reasoning activities during the intervention sessions. These collaborative processes enabled learners to articulate ideas, compare perspectives, identify errors, and construct shared understanding through interaction with classmates. Such findings correspond with those of Chien et al., who reported that collaborative STEM-based learning environments significantly enhance students' reasoning abilities, creativity, and engagement through interactive problem-solving experiences (11). Therefore, collaborative interaction appears to play an essential role in facilitating both mathematics achievement and critical thinking development.

Another significant aspect of the present findings was the positive influence of real-world contextualization on students' learning experiences. The instructional intervention incorporated authentic mathematical problems connected to daily life situations, enabling students to recognize the practical relevance of mathematical concepts. Classroom observations indicated that students demonstrated greater motivation and deeper engagement when mathematical problems were linked to realistic contexts and meaningful applications. These findings are consistent with those reported by Han et al., who emphasized that integrating real-world contexts into mathematics instruction strengthens conceptual understanding and enhances learners' problem-solving abilities (5). Similarly, Adelabu and Pharamela highlighted the importance of teaching relevant mathematical topics capable of preparing students for practical and professional challenges beyond the classroom (7). The findings therefore suggest that contextualized mathematics instruction may increase both cognitive engagement and educational relevance.

The study also demonstrated the importance of reflective thinking and self-evaluation in mathematics learning. Students participating in the integrated instructional program were encouraged to analyze their mistakes, revise their reasoning processes, and critically evaluate alternative solutions. These reflective practices appeared to strengthen students' metacognitive awareness and analytical reasoning abilities. Reflective thinking allows learners to monitor their cognitive processes, identify conceptual misunderstandings, and develop more effective problem-solving strategies. Rezaee and Saleh similarly emphasized that critical thinking development depends on the incorporation of reflective and analytical dimensions within instructional frameworks and educational materials (19). The findings of the present study therefore reinforce the importance of metacognitive and reflective processes in promoting meaningful mathematics learning.

The role of teachers emerged as another critical factor influencing the effectiveness of integrated critical thinking instruction. The qualitative findings indicated that teachers who employed inquiry-oriented and facilitative instructional approaches created more interactive and cognitively stimulating classroom environments. These teachers encouraged questioning, discussion, exploration, and collaborative reasoning rather than focusing exclusively on procedural explanation. Such instructional behaviors appeared to contribute substantially to students' engagement and analytical development. Habiyaremye et al. emphasized that effective mathematics teaching requires strong pedagogical content knowledge and the ability to facilitate competency-based and student-centered learning experiences (15). Likewise, Saleh and AlAli argued that mathematics teacher preparation programs should equip educators with the pedagogical skills necessary for implementing active learning and critical inquiry in mathematics classrooms (8). Consequently, teacher competence and instructional philosophy appear to be central determinants of successful critical thinking integration.

Technological and digital dimensions of mathematics education also provide important implications for interpreting the present findings. Although the intervention in the current study primarily emphasized classroom-based instructional interaction, the findings align with contemporary perspectives highlighting the role of educational technologies in supporting critical thinking and analytical reasoning. Zhou et al. demonstrated that AI-supported scaffolding systems can improve students' mathematical problem-solving performance by facilitating reflection, guidance, and adaptive feedback during learning activities (12). Similarly, Heidarpour et al. emphasized that artificial intelligence technologies can support the development of soft skills and higher-order thinking abilities when integrated appropriately into educational environments (13). Furthermore, Uwineza et al. reported that interactive mathematics software positively influences learners' mathematics achievement and classroom engagement by promoting dynamic and student-centered learning experiences (14). These findings collectively suggest that future instructional models integrating critical thinking may benefit from combining learner-centered pedagogies with digital and AI-supported educational tools.

The findings of the present study additionally support broader educational reform perspectives emphasizing curriculum innovation and competency-based education. Matorevhu argued that curriculum innovation aimed at preparing students for industrialization and modern workforce demands requires educational systems to prioritize critical thinking, adaptability, and problem-solving competencies (16). Similarly, Wade et al. emphasized the necessity of developing instructional models capable of supporting students' transition toward more advanced mathematical reasoning and conceptual understanding (21). The integrated instructional model examined in the present study appears consistent with these broader educational objectives because it emphasizes analytical reasoning, inquiry, reflection, collaboration, and practical problem-solving abilities rather than narrow procedural performance.

The present findings also highlight the importance of reconsidering traditional educational paradigms that limit students' opportunities for intellectual exploration and critical inquiry. Owens argued that mathematics education should move beyond restrictive and narrow conceptions of mathematical learning toward more inclusive and intellectually empowering approaches capable of fostering reflective and analytical thinking (6). Similarly, Rukundo discussed how educational systems influenced by rigid traditional structures may inhibit the development of critical and independent thinking abilities among learners (17). The results of the current study suggest that integrated critical thinking instruction may provide an effective strategy for overcoming such limitations by creating more participatory, inquiry-oriented, and intellectually engaging mathematics classrooms.

Finally, the findings support Talanquer et al.'s systems-oriented perspective emphasizing that effective learning emerges from the interaction of multiple interconnected educational components, including instructional design, classroom interaction, learner cognition, and sociocultural context (18). The success of the integrated instructional intervention in the present study appears to have resulted not from a single instructional technique but from the coordinated combination of collaborative learning, inquiry-based activities, reflective reasoning, contextualized problems, teacher facilitation, and active student engagement. Therefore, improving mathematics performance and critical thinking development requires comprehensive and multidimensional instructional frameworks rather than isolated pedagogical modifications.

One of the limitations of the present study was the restriction of the sample to tenth-grade students in secondary schools located in Tehran, which may limit the generalizability of the findings to other educational levels, geographical regions, or cultural contexts. Furthermore, the intervention period consisted of only eight instructional sessions, and a longer implementation period might have produced more substantial or stable effects. Another limitation involved the reliance on self-report questionnaires for measuring critical thinking, which may be influenced by participants' subjective perceptions and response tendencies. Additionally, classroom conditions, teacher differences, and school resources may have affected the implementation process and students' learning experiences.

Future research is recommended to examine the effectiveness of integrated critical thinking instruction across different educational levels, academic disciplines, and sociocultural settings. Researchers may also investigate the long-term effects of such instructional models on students' academic achievement, cognitive flexibility, and problem-solving abilities through longitudinal studies. Further studies could explore the integration of artificial intelligence tools, digital learning platforms, and interactive technologies into critical thinking-oriented mathematics instruction. Comparative studies examining different instructional approaches and teacher training models may also provide valuable insights into the most effective strategies for promoting higher-order thinking in mathematics education.

The findings of the present study have several practical implications for educational policymakers, curriculum developers, school administrators, and mathematics teachers. Educational systems should prioritize the integration of critical thinking components into mathematics curricula and instructional frameworks in order to improve students' analytical reasoning and conceptual understanding. Teacher preparation and professional development programs should provide educators with practical training in inquiry-based learning, collaborative instruction, reflective teaching, and critical thinking facilitation. Schools should also create supportive classroom environments encouraging active participation, questioning, and collaborative problem solving. Furthermore, educational planners should consider integrating real-world applications, digital technologies, and learner-centered instructional methods into mathematics education in order to enhance both academic performance and higher-order cognitive development among students.

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Authors' Contributions

All authors equally contributed to this study.

Declaration of Interest

The authors of this article declared no conflict of interest.

Ethical Considerations

All ethical principles were adhered in conducting and writing this article.

Transparency of Data

In accordance with the principles of transparency and open research, we declare that all data and materials used in this study are available upon request.

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